

Effective dark matter fluid with higher derivative corrections.

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Abstract

The effective field theory for hydrodynamics allows to write the action functional for fluid. In this paper, some simplest possible higher derivative terms in the fluid action and the cosmological consequences of their presence are considered. Particular attention is given to dark matter, modelled as a dust with higher derivative corrections. We study the conditions of absence of singularities in the solutions of the background and perturbed equations and investigate the evolution of perturbations in two simple models of matter dominated Universe. There is a range of parameters describing the higher derivative terms, in which the short-wavelength perturbations of dark matter are suppressed and the dark matter can be seen as fairly homogeneous on a sufficiently small scale.

1 Introduction.

Relativistic fluids play an important role in many areas of modern cosmology and astrophysics. They are widely used for the description of various kinds of cosmological species. As an example, the standard Big Bang cosmological model (Λ CDM) contains non-clustering dark energy in the form of a small positive cosmological constant Λ together with cold dark matter considered as a perfect fluid with zero pressure. The standard model fits well to the supernovae data [1], Baryon Acoustic Oscillations surveys [2] and measurements of the cosmic microwave background [3, 4]. More complicated models are also developed to avoid such theoretical problems of Λ CDM cosmology as fine-tuning and coincidences problems. These are primarily the models with dynamic dark energy (see [5] for a review) and cosmological models on the basis of modified theories of gravity (a recent review can be found in [6]). Generalized models of dark matter alone are also of interest.

There are several different approaches to cosmological perfect fluids. A promising way of describing a perfect relativistic fluid is outlined by the pull-back formalism [7] and the

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effective field theory for hydrodynamics [8]. In these closely related techniques, a barotropic perfect fluid can be described by a set of three non-canonical scalar fields satisfying certain symmetry conditions [9]. The advantage of these formalisms is the ability to write a perfect fluid action in simple and convenient form that offers great possibilities for generalizations. For example, the effective field theory allows to describe a charged fluid [10]. Currently, both approaches are evolving to account for a range of dissipative effects [11, 12].

Cosmological applications of the variational principle for fluids (and solids) were first considered in [13, 14, 15, 16]. In [14] the corresponding formalism has been used to constrain possible deviations from Lorentz invariance in dark matter. It was applied to propose and investigate new classes of inflationary models [15] as well as new form of possible coupling between dark matter and quintessence [17]. The cosmological effective field theory of multi-component fluids has been considered in [18]. The Brown’s formulation of the variational principle for relativistic fluids (with Lagrangian multipliers) [19] currently is intensively employed in new multi-fluid [20] and Scalar-fluid theories [21, 22, 23].

The aim of this paper is to investigate some cosmological effects of higher derivative corrections to the action of perfect fluid within the framework of the effective field theory. A sample of such corrections has been considered in the Minkowski background in [10]. In a cosmological context, the effective theory of fluids at next-to-leading order in derivatives and implications for dark energy have been explored in [24]. This paper is organized as follows. In Section 2, we review the formalism used in the analysis and present our model. The background solutions are investigated in Section 3. The Section 4 contains the perturbed Einstein equations for scalar linear perturbations about spatially flat FLRW background. We examine also the conditions of the absence of instabilities in radiation-dominated and matter-dominated eras. In Section 5, the evolution of perturbations is studied in the toy model and, by numerical methods, in a more realistic model with baryonic matter. We conclude in Section 6.

2 Setup

Let us briefly review some basic concepts and results of effective field theory for hydrodynamics. Initially, this formalism has been developed as the theory of phonons in a continuous medium [25, 26, 27, 9]. It has been applied also to rederiving, in a field theory language, some findings of the pull-back formalism [28, 29], which is based on earlier formulations of the variational principle for fluids [30, 31]. Here we mainly follow the papers [9, 10, 16].

In continuum mechanics, the central role plays the concept of fluid element. A fluid element is always situated at the same fluid point moving with the velocity of the flow [32]. The fluid points can be marked with three labels φ^a ($a = 1, 2, 3$), which are usually chosen to be a fluid point coordinates at some initial time. At any time t the coordinates of the fluid element are given by $x^i = x^i(t, \varphi^a)$. This equation thus defines the trajectory of the fluid element and corresponds to a Lagrangian description of the fluid motion.

The inverse mapping

$$x^i \rightarrow \varphi^a(t, \mathbf{x}) \tag{1}$$

gives the labels of the fluid point that is located at spatial point \mathbf{x} at the moment t . In other words, the mapping (1) provides an Eulerian representation of fluid flow. Once the

labels are being attached to the fluid points, they do not change under space-time coordinate transformations. Hence, the functions $\varphi^a(t, \mathbf{x})$ are scalar fields from the observer's point of view.

The three-dimensional space of fluids points and the spatial space are isomorphic, that allows to consider the map (1) as the transformation to a coordinate system in which the fluid is at rest. Therefore, the fields $\varphi^i(t, \mathbf{x})$ can be treated as the comoving coordinates of the fluid. This leads to the orthogonality condition ($u^\mu = dx^\mu/d\eta$, where η is the proper time)

$$u^\mu \partial_\mu \varphi^a = 0. \quad (2)$$

The action of any physical theory which uses fields $\varphi^a(t, \mathbf{x})$ should not depend on the ambiguity of its definition. In particular, the action of the system should not be changed within internal shifts and rotations:

$$\varphi^a \rightarrow \varphi^a + c^a, \quad (3)$$

$$\varphi^a \rightarrow O^a_b \varphi^b, \quad O \in SO(3), \quad (4)$$

where c^a and the matrix entries O^a_b are constant.

In addition, to distinguish a fluid from an isotropic solid, we require the invariance under volume preserving transformations

$$\varphi^a \rightarrow f^a(\varphi), \quad \det \left(\frac{\partial f^a}{\partial \varphi^b} \right) = 1. \quad (5)$$

2.1 Barotropic perfect fluid.

The invariance under internal translations (3) implies that the Lagrangian is a function of the quantities $\varphi^a_{,\mu} \equiv \partial_\mu \varphi^a$ and their partial derivatives. At lowest order in the derivative expansion, it will involve only one derivative acting on each field φ^a . Then the Lorentz invariance leads to the fact that the lowest order Lagrangian should be built from the set of scalar quantities

$$B^{ab} \equiv B_{ab} = g^{\mu\nu} \varphi^a_{,\mu} \varphi^b_{,\nu}, \quad (6)$$

that form a matrix B .

There is only one independent function of entries of any 3×3 matrix, namely its determinant, which is invariant under the internal transformations (4), (5) [15]. Hence, the low-order action consistent with the imposed symmetries is

$$S_{(bf)} = \int F(b) \sqrt{-g} d^4x, \quad (7)$$

where F is a generic function and

$$b = \sqrt{\det B}. \quad (8)$$

From the viewpoint of field theory, this action is interpreted as a low-energy one [8].

The orthogonality conditions (2) can be resolved to give [9, 10]

$$u^\mu = \frac{1}{b\sqrt{-g}} \epsilon^{\mu\alpha\beta\gamma} \varphi^1_{,\alpha} \varphi^2_{,\beta} \varphi^3_{,\gamma}, \quad (9)$$

where the Levi-Civita symbol ϵ is defined by $\epsilon^{0123} = +1$, and we use the usual normalization

$$u^\mu u_\mu = -1. \quad (10)$$

It is worth noting that the four-velocity u^μ is invariant to an arbitrary internal diffeomorphism [8].

We define the energy-momentum tensor as

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}}, \quad (11)$$

that gives for the low-order action (7)

$$T_{\mu\nu(bf)} = F g_{\mu\nu} - F_{,b} b B^{-1}_{ab} \varphi^a_{,\mu} \varphi^b_{,\nu}. \quad (12)$$

In view of equations (2) and (9), this is the energy-momentum tensor of a perfect fluid

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu}, \quad (13)$$

whose energy density and pressure are given by [9]

$$\rho = -F, \quad p = F - F_{,b} b. \quad (14)$$

Hence, the action (7) describes a barotropic perfect fluid with the equation of state

$$p = w\rho, \quad (15)$$

where

$$w \equiv \frac{p}{\rho} = \frac{F_{,b}}{F} b - 1. \quad (16)$$

The dust has zero pressure $p = 0$, that leads to $F \propto b$. The radiation fluid is described by $F \propto b^{4/3}$. Generally, the power law dependence of the function $F(b)$ corresponds to the case of barotropic fluid with a constant parameter of state w .

Within the reviewed above formalism there is conserved current

$$J^\mu = b u^\mu. \quad (17)$$

This vector field satisfies the identity [10, 16]

$$J^\mu_{;\mu} \equiv 0. \quad (18)$$

where semicolon indicates covariant differentiation. The quantities J^μ and b can be identified with the entropy current and the entropy density correspondingly [10].

2.2 Higher derivative terms.

The theory outlined above has been purely classical. The classical treatment of the effective field theory should be applicable on scales of the order of $1/\Lambda$ or longer, where Λ is some positive constant. We assume that a cut-off parameter Λ is smaller than the Planck mass $M_{PL} \equiv 1/\sqrt{8\pi G}$, but large enough to cover all relevant cosmological scales. To be able to neglect quantum fluctuations, the scalar fields φ^a would be sufficiently smooth [9]:

$$|\partial_\nu \varphi^a| \gg \frac{\partial_\mu \partial_\nu \varphi^a}{\Lambda}. \quad (19)$$

This inequality allows us to use a derivative expansion and write the action of the theory in the form [9, 24]

$$S = \int \Lambda^4 \tilde{F} \left(\epsilon \frac{\varphi^a}{\Lambda}, \frac{B^{ab}}{\Lambda^4}, \frac{\varphi^{a;\mu} \varphi^{b;\nu}}{\Lambda^6}, \dots \right) \sqrt{-g} d^4 x, \quad (20)$$

where $\epsilon \ll 1$ is a small dimensionless parameter. It is assumed usually that all dimensionless parameters (except ϵ) in the function \tilde{F} are of order one. This last assumption is very strong in some cases, we are not imposing it here. Nevertheless, we expect that a derivative expansion of \tilde{F} is uniformly convergent at sufficiently smooth φ^a . In what follows we will work in units where $\Lambda = c = \hbar = 1$.

Consider now possible corrections to the Lagrangian of a barotropic perfect fluid. We will constraint the great variety of possible additional terms by the assumption that our field theory will contain no derivatives of scalar fields φ^a higher than second order. Within the framework of fluid description, such terms can be constructed from hydrodynamic variables ρ , u^μ and their first order derivatives. Following [10], we will group the corrections according to the total number of involved derivatives of the hydrodynamic variables.

The most general first-order term in the Lagrangian which is compatible with the symmetries (3)-(5) has the form

$$f(b) b_{,\mu} u^\mu, \quad (21)$$

where $f(b)$ is an arbitrary function. It is worth noting that this term is linear in the second derivatives of fields φ^a . Since the equation (18) is an identity, the theory is free of first-order corrections [10, 16]. Indeed, in view of the equations (17) and (18), the term (21) can be reduced to a total derivative.

The general form of second-order correction is a linear combination of six scalars [33, 24]

$$\begin{aligned} & h_1(b) (u^\mu b_{,\mu})^2, \quad h_2(b) b^\mu b_{,\mu}, \quad h_3(b) u^\mu_{;\nu} u^\nu_{;\mu}, \\ & h_4(b) \varepsilon_{\alpha\beta\mu\nu} u^{\alpha;\beta} u^{\mu;\nu}, \quad h_5(b) (u^\mu_{;\mu})^2, \quad h_6(b) u^\mu u^\nu u^\alpha_{;\mu} u_{\alpha;\nu}, \end{aligned} \quad (22)$$

where $h_1(b), \dots, h_6(b)$ are arbitrary scalar functions, and $\varepsilon_{\alpha\beta\mu\nu}$ is the alternating unit tensor. Fluid Lagrangians involving the term $\gamma (u^\mu_{;\mu})^2$ with constant γ have been examined recently [34, 35] in the context of mimetic gravity [36]. The more general models, in which γ is a function of the mimetic field, are also investigated in [35].

The case of second-order correction

$$f(b) b_{,\mu} b^{,\mu} \quad (23)$$

appears to be the simplest to general study. A sample of such term with $f(b) = \text{const}$ has been considered in [10]. Moreover, the term (23) plays a crucial role in the analysis of dark matter perturbations in the framework of the effective field theory for hydrodynamics (we will discuss this issue at the beginning of Section 4).

The second-order term (23) in the Lagrangian gives the following contribution to the energy-momentum tensor

$$T_{\mu\nu(hd)} = -2fb_{,\mu}b_{,\nu} + fb_{,\sigma}b^{;\sigma}g_{\mu\nu} + (2fbb^{;\sigma}{}_{;\sigma} + f_{,b}b^{;\sigma}b_{;\sigma}b)(g_{\mu\nu} + u_\mu u_\nu). \quad (24)$$

The variation of the total action with respect to scalar fields φ^a yields the equations of motion of the fluid

$$\left[\sqrt{|g|}g^{\mu\nu}b(F_{,b} - f_{,b}b_{,\sigma}b^{;\sigma} - 2fbb^{;\sigma}{}_{;\sigma})(B^{-1})_{ab}\varphi^a_{,\mu} \right]_{,\nu} = 0. \quad (25)$$

2.3 Model.

In the following, we will study the case of the power-law dependence of $f(b)$. The corresponding term in the Lagrangian takes the form

$$\alpha b^n b_{,\mu} b^{;\mu}. \quad (26)$$

This choice of second-order corrections looks natural since the description of any perfect fluid with constant equation of state also uses the power-law dependence of the function $F(b)$.

To completely specify a model, one need to choose the lowest order Lagrangian. In this paper, we consider a model of cold dark matter with higher derivative corrections performing a more general analysis where it is convenient or necessary. The cold dark matter action with considered higher derivative corrections has the form

$$S = \int (\alpha b^n b_{,\mu} b^{;\mu} + \beta b) \sqrt{|g|} d^4x, \quad (27)$$

where α and β are some constants. Instead the coupling constant α , one can use the dimensionless parameter $8\pi G\alpha$.

3 Background.

In this Section we consider a homogeneous, isotropic and spatially flat Universe. The line element has the form

$$ds^2 = a^2 (-d\tau^2 + \delta_{ij}x^i x^j), \quad (28)$$

where a is the scale factor and τ is conformal time.

Since the fields φ^a can be regarded as comoving coordinates of the fluid, one can write

$$\bar{\varphi}^a = x^a. \quad (29)$$

Here, the bar indicates that the expression is evaluated for the background line element (28).

Substituting the background fields $\bar{\varphi}^a$ into the equation (6), we obtain [16]

$$\bar{B}_{ab} = \frac{1}{a^2} \delta_{ab}, \quad \bar{b} = \frac{1}{a^3}. \quad (30)$$

A direct check shows that the background fields $\bar{\varphi}^a$ are the solutions of the field equations (25).

To study the background evolution of the fluid with higher derivative corrections, we use a slightly different approach than that was implemented in [24]. We split a full energy-momentum tensor into two parts:

$$T_{\mu\nu} = T_{\mu\nu(m)} + T_{\mu\nu(hd)}, \quad (31)$$

where $T_{\mu\nu(hd)}$ is defined by the equation (24). When, the equations (31) and (24) give the following expressions for the background energy-momentum tensor components

$$\bar{T}_0^0 = -\bar{\rho} + 9\alpha \frac{\mathcal{H}^2}{a^2} a^{-3(n+2)}, \quad (32)$$

$$\bar{T}_j^i = \left[\bar{p} + 3\alpha \frac{\mathcal{H}^2}{a^2} a^{-3(n+2)} \left(2 \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - 3n - 5 \right) \right] \delta_j^i, \quad (33)$$

where $\mathcal{H} = \dot{a}/a$, the dot denote the derivative respect to conformal time τ , and

$$\bar{\rho} = -\bar{T}_{0(m)}^0, \quad (34)$$

$$\bar{p} = \bar{T}_{j(m)}^i \quad (i = j). \quad (35)$$

The equations (32), (33) indicate that the background energy-momentum tensor depends on the quantity \mathcal{H} and its derivative with respect to the conformal time. The energy-momentum tensors with the same feature have been studied in cosmological models with a viscous fluid (recent review can be found in [37]).

The dynamics of the scale factor determined by the background Einstein equations

$$\bar{G}^\mu_\nu = 8\pi G \bar{T}^\mu_\nu, \quad (36)$$

where \bar{G}^μ_ν is the background Einstein tensor and G is the gravitational constant. The non-zero components of the Einstein tensor for spatially flat metric are

$$\bar{G}_0^0 = -3 \frac{\mathcal{H}^2}{a^2}, \quad \bar{G}_1^1 = \bar{G}_2^2 = \bar{G}_3^3 = -\frac{2\dot{\mathcal{H}} + \mathcal{H}^2}{a^2}. \quad (37)$$

It is convenient to introduce the notation

$$\tilde{G}(a) = \frac{G}{1 + 24\pi G \alpha a^{-3(n+2)}}. \quad (38)$$

In general, the quantity \tilde{G} is a function of the scale factor a . This function is constant at $n = -2$.

Now the Einstein equations lead to

$$\mathcal{H}^2 = \frac{8\pi\tilde{G}}{3}a^2\bar{\rho}, \quad (39)$$

$$\dot{\mathcal{H}} = -\frac{4\pi\tilde{G}}{3}a^2\bar{\rho} \left[(1+3w) - \frac{72\pi\tilde{G}\alpha}{a^{3(n+2)}}(n+2) \right]. \quad (40)$$

The equation (39) allows to interpret the quantity \tilde{G} as the effective background gravitational "constant". In this interpretation, any theory with the term (26) can be considered as a modified theory of gravity. At $n = -2$, the equations (39), (40) are reduced to the corresponding Friedmann equations with a renormalized gravitational constant.

For negative α , the quantities \tilde{G} and \mathcal{H} can be infinite. It occurs when the scale factor takes the critical value a_s , which is the solution of the equation

$$\frac{24\pi G\alpha}{a_s^{3(n+2)}} = -1. \quad (41)$$

The critical value a_s is determined by the coupling constant α and the power-law index n . At $n > -2$ and sufficiently small $|\alpha|$, the singularity falls on the Planck era near which our effective field theory is not applicable. This singularity also can be avoided in more complex models that involve a generating process of the dark matter fluid. At $n < -2$, the role of term (26) increases with time and the singularity can be eliminated in models with a decaying fluid.

For positive α , it is possible to have

$$\frac{24\pi G\alpha}{a^{3(n+2)}} \gg 1. \quad (42)$$

This regime realized for sufficiently small values of the scale factor when $n > -2$, or for sufficiently large values of the scale factor when $n < -2$. In this mode, we obtain $\tilde{G}/G \approx a^{3(n+2)}/(24\pi G\alpha)$, and the equation (39) is significantly different from the usual second Friedman equation. However, the near-standard evolution of the scale factor $a(\tau)$ during the radiation dominated and matter dominated stages can be obtained only when the influence of higher derivative terms on the background solution is weak or veiled.

Hence, the treatment of only one type (26) of corrections is applicable if we have $n \approx -2$ with a good accuracy, or if the background effect of these corrections is subdominant during considered time period

$$\frac{24\pi G|\alpha|}{a^{3(n+2)}} \ll 1. \quad (43)$$

4 Perturbations.

To begin with, let us consider the perturbations of the fields φ^a in the Minkowski space-time. We use here the parametrization [9, 16, 24]

$$\varphi^a = x^a + \pi^a. \quad (44)$$

By expanding the equations (6), (8), (9) up to second order in π , one can obtain [24]

$$b = 1 + \partial_i \pi^i - \frac{1}{2} \dot{\pi}^2 - \frac{1}{2} \partial_i \pi^j \partial_j \pi^i + \frac{1}{2} (\partial_i \pi^i)^2 \quad (45)$$

and

$$u^0 = 1 + \frac{1}{2} \dot{\pi}^2, \quad u^i = -\dot{\pi}^i + \dot{\pi}^k \partial_k \pi^i. \quad (46)$$

For simplicity, we assume that the wavelengths of the perturbations are long enough and $\bar{\rho} \neq 0$. Then the quadratic action for dark matter perturbations, including *all* higher derivative terms (22), takes the form

$$S^{(2)} = S_{LO}^{(2)} + S_{NLO}^{(2)}, \quad (47)$$

where ¹

$$S_{LO}^{(2)} = \frac{\bar{\rho}}{2} \int (\dot{\pi}_L^2 + \dot{\pi}_T^2) d^4x, \quad (48)$$

$$S_{NLO}^{(2)} = h_2 \int (\partial_j (\partial_i \pi^i))^2 d^4x, \quad (49)$$

and we split the fields π^i into the longitudinal and transverse parts:

$$\pi^i = \pi_L^i + \pi_T^i, \quad \partial_i \pi_T^i = 0, \quad \epsilon_{ijk} \partial_j \pi_L^k = 0. \quad (50)$$

The variation of the quadratic action (47) with respect to the perturbations π^i gives the equations of motion

$$\ddot{\pi}_T^i = 0, \quad (51)$$

$$\ddot{\pi}_L^i - 2 \frac{h_2}{\bar{\rho}} \partial^2 (\partial_k \partial_i \pi_L^k) = 0. \quad (52)$$

The latter equation is conveniently rewritten using a new variable s defined by [24]

$$\pi_L^i = \partial_i s. \quad (53)$$

The equation (52) yields in Fourier space

$$\ddot{s} - 2 \frac{h_2}{\bar{\rho}} k^4 s = 0. \quad (54)$$

Hence, longitudinal perturbations do not grow exponentially with time when

$$\frac{h_2}{\bar{\rho}} \leq 0. \quad (55)$$

For the model (27) it gives the constraint

$$\alpha \leq 0. \quad (56)$$

Under the condition (55), the equation (54) describe a simple harmonic oscillator with k -dependent mass term which stem from higher derivative corrections. Consequently, in the effective field theory approach for dark matter, we expect $\dot{\pi} \sim \sqrt{h_2/\bar{\rho}} k^2 \pi$ and $S_{NLO}^{(2)} \sim S_{LO}^{(2)}$ due the equations of motion.

¹The equation (49) is exactly the same as the Eq. (3.20) of [24] for considered here case $c_\pi^2 \equiv \bar{p}'/\bar{\rho}' = 0$, $\bar{p} = 0$.

4.1 Perturbed Einstein equations.

Let us consider small linear perturbations around a spatially flat Friedmann-Robertson-Walker Universe, focusing on scalar perturbations. It is convenient to use the freedom of gauge transformations and fix the longitudinal gauge. The line element in the longitudinal gauge has the form [38]

$$ds^2 = a^2(\tau) \left\{ -(1 + 2\Phi)d\tau^2 + (1 - 2\Psi)\delta_{ij}dx^i dx^j \right\}, \quad (57)$$

where Φ and Ψ are some small scalar quantities which are functions of space and time coordinates.

The scalar fields φ^a also can be split into a background part and a small perturbation

$$\varphi^a(\tau, x^i) = \bar{\varphi}^a(\tau, x^i) + \delta\varphi^a(\tau, x^i). \quad (58)$$

Since $\bar{\varphi}^i(\tau, x^i) = x^i$, the equations (6), (8) yield [16]

$$b = \frac{1}{a^3} (1 + 3\Psi + \delta\varphi^k{}_{,k}). \quad (59)$$

The equation (9) gives

$$u^0 = \frac{1}{a} (1 - \Phi), \quad u^i = -\frac{1}{a} \delta\dot{\varphi}^i. \quad (60)$$

We consider only the scalar perturbations. This means that, in particular, one can introduce a velocity potential v such that $u^i = v^{,i}/a$. The equation (60) then leads to

$$\delta\varphi^i = \delta\chi^{,i}, \quad v = -\delta\dot{\chi}. \quad (61)$$

where $\delta\chi$ is some small scalar quantity.

The linearization of the energy-momentum tensor of relativistic fluid with second-order corrections yields

$$\delta T^0_0 = -\delta\rho + 6f\mathcal{H}^2 \frac{\bar{b}^2}{a^2} \left[3\frac{\delta b}{\bar{b}} - 3\Phi - \frac{1}{\mathcal{H}} \left(\frac{\delta b}{\bar{b}} \right)^{\cdot} + \frac{3}{2} \frac{f_{,b}}{f} \delta b \right], \quad (62)$$

$$\delta T^0_i = (\bar{\rho} + \bar{p})v - 6f\mathcal{H}^2 \frac{\bar{b}^2}{a^2} \left[\frac{1}{\mathcal{H}} \left(\frac{\delta b}{\bar{b}} \right)_{,i} - \left(1 - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{3}{2} \frac{f_{,b}}{f} \bar{b} \right) \delta\dot{\varphi}_i \right], \quad (63)$$

$$\begin{aligned} \delta T^i_j = & \delta\bar{p}\delta^i_j + 2f\frac{\bar{b}^2}{a^2} \left[-\left(\frac{\delta b}{\bar{b}} \right)^{\cdot\cdot} + \mathcal{H} \left(7 + 3\frac{f_{,b}}{f} \bar{b} \right) \left(\frac{\delta b}{\bar{b}} \right)^{\cdot} \right. \\ & + \mathcal{H}^2 \left(3 \left(2\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - 5 \right) + 3\frac{f_{,b}}{f} \bar{b} \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - 7 \right) - \frac{9}{2} \frac{f_{,bb}}{f} \bar{b}^2 \right) \frac{\delta b}{\bar{b}} \\ & \left. + \nabla^2 \frac{\delta b}{\bar{b}} + 3\mathcal{H}^2 \left(5 - 2\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + 3\frac{f_{,b}}{f} \bar{b} \right) \Phi - 3\mathcal{H} (3\Psi + \dot{\Phi}) \right] \delta^i_j, \end{aligned} \quad (64)$$

where we denote

$$\delta\rho = -\bar{F}_{,b}\delta b, \quad \delta p = -\bar{b}\bar{F}_{,bb}\delta b. \quad (65)$$

Using equations (59) and (62)-(64), one can write the perturbed Einstein equations [38]. For the case $f = \alpha b^n$, they are

$$3\mathcal{H}(\dot{\Phi} + \mathcal{H}\Phi) - \nabla^2\Phi = -4\pi G a^2 \delta\rho - \frac{24\pi G\alpha}{a^{3(n+2)}} \left[3\mathcal{H}(\dot{\Phi} + \mathcal{H}\Phi) + \mathcal{H}\nabla^2\delta\dot{\chi} - \frac{3}{2}\mathcal{H}^2(n+2)(3\Phi + \nabla^2\delta\chi) \right], \quad (66)$$

$$\dot{\Phi} + \mathcal{H}\Phi = -4\pi G a^2 (\bar{\rho} + \bar{p})v - \frac{24\pi G\alpha}{a^{3(n+2)}} \left[\frac{1}{2} \left(3n+2 - 2\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} \right) \mathcal{H}^2\delta\dot{\chi} - \mathcal{H}(3\Phi + \nabla^2\delta\chi) \right], \quad (67)$$

$$\ddot{\Phi} + 3\mathcal{H}\dot{\Phi} + (2\dot{\mathcal{H}} + \mathcal{H}^2)\Phi = 4\pi G\delta p - \frac{24\pi G\alpha}{a^{3(n+2)}} \left[3\ddot{\Phi} - 9(1+n)\mathcal{H}\dot{\Phi} + \left(6 + \frac{9}{2}n \right) \left(3n+5 - 2\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} \right) \mathcal{H}^2\Phi - 3\nabla^2\Phi + \nabla^2 \left(\delta\ddot{\chi} - (7+3n)\mathcal{H}\delta\dot{\chi} + \frac{3}{2}(n+2) \left(3n+5 - 2\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} \right) \mathcal{H}^2\delta\chi - \nabla^2\delta\chi \right) \right]. \quad (68)$$

Here we used the equality $\Psi = \Phi$ which follows from the fact that the spatial part of the energy-momentum tensor is diagonal. In this form, the perturbed Einstein equations can be easily generalized to the case of multiple minimally coupled fluids with vanishing anisotropic stresses.

The higher derivative terms may cause singularities in the solution of the perturbed equations. These singularities can be avoided by tuning the model parameters or initial conditions. For the rest of this section we investigate the constraints on the parameters, which follow from the condition of the absence of instabilities during the radiation dominated and matter dominated stages.

4.2 Instabilities in the dust dominated Universe.

Consider a Universe filled with ordinary non-relativistic matter (b) and cold dark matter with higher derivative corrections (c). For our purposes, the most convenient equations are the Einstein equations (66), (67), in which we have to make the substitution

$$\delta\rho = \delta\rho_b - \beta\delta b, \quad (69)$$

$$(\bar{\rho} + \bar{p})v = \bar{\rho}_b v_b + \beta\bar{b}\delta\dot{\chi}. \quad (70)$$

Since

$$\bar{\rho}_c \equiv -\beta\bar{b}, \quad (71)$$

the equation (70) takes the form of the well-known expression for the case of multi-component fluid

$$(\bar{\rho} + \bar{p})v = \sum_A (\bar{\rho}_A + \bar{p}_A)v_A, \quad (72)$$

where subscript A denotes the fluid species.

The equations (30), (39), (71) lead to

$$4\pi G \frac{\beta}{a} = -\frac{3}{2} \frac{G}{\tilde{G}} \frac{\bar{\rho}_c}{\bar{\rho}} \mathcal{H}^2. \quad (73)$$

In addition, there is the equality

$$\bar{\rho} = \bar{\rho}_b + \bar{\rho}_c. \quad (74)$$

The independent perturbed Einstein equations can be written now in the Fourier space as

$$\begin{aligned} \dot{\Phi} - \frac{8\pi\tilde{G}\alpha}{a^{3(n+2)}} k^2 \delta\dot{\chi} = & -\frac{1}{2} \frac{\rho_b}{\bar{\rho}} \mathcal{H} \delta_b + \left(\frac{1}{2} \frac{\bar{\rho}_c}{\bar{\rho}} - \frac{12\pi\tilde{G}\alpha}{a^{3(n+2)}} (n+2) \right) \mathcal{H} k^2 \delta\chi \\ & - \left(1 + \frac{3}{2} \frac{\bar{\rho}_c}{\bar{\rho}} + \frac{k^2}{3\mathcal{H}^2} \frac{\tilde{G}}{G} - \frac{36\pi\tilde{G}\alpha}{a^{3(n+2)}} (n+2) \right) \mathcal{H} \Phi, \end{aligned} \quad (75)$$

$$\begin{aligned} \dot{\Phi} - \left(\frac{3}{2} \frac{G}{\tilde{G}} \frac{\bar{\rho}_c}{\bar{\rho}} - \frac{12\pi G\alpha}{a^{3(n+2)}} \left(3n+2 - 2 \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} \right) \right) \mathcal{H}^2 \delta\dot{\chi} = \\ - \frac{3}{2} \frac{G}{\tilde{G}} \frac{\bar{\rho}_b}{\bar{\rho}} \mathcal{H}^2 v_b - \frac{24\pi G\alpha}{a^{3(n+2)}} \mathcal{H} k^2 \delta\chi - \left(1 - \frac{72\pi G\alpha}{a^{3(n+2)}} \right) \mathcal{H} \Phi. \end{aligned} \quad (76)$$

These equations can be solved with respect to $\delta\dot{\chi}$ and $\dot{\Phi}$. For example, subtracting (76) from (75), we obtain

$$\begin{aligned} \left(\frac{3}{2} \frac{G}{\tilde{G}} \frac{\bar{\rho}_c}{\bar{\rho}} - \frac{12\pi G\alpha}{a^{3(n+2)}} \left(3n+2 - 2 \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} \right) - \frac{8\pi\tilde{G}\alpha}{a^{3(n+2)}} \frac{k^2}{\mathcal{H}^2} \right) \mathcal{H} \delta\dot{\chi} = \frac{3}{2} \frac{G}{\tilde{G}} \frac{\bar{\rho}_b}{\bar{\rho}} \mathcal{H} v_b \\ - \frac{1}{2} \frac{\rho_b}{\bar{\rho}} \delta_b - \frac{k^2}{3\mathcal{H}^2} \frac{\tilde{G}}{G} \Phi - \left(\frac{1}{2} \frac{\bar{\rho}_c}{\bar{\rho}} + \frac{24\pi G\alpha}{a^{3(n+2)}} - \frac{12\pi\tilde{G}\alpha}{a^{3(n+2)}} (n+2) \right) (3\Phi - k^2 \delta\chi). \end{aligned} \quad (77)$$

The equation (77) indicates that the conformal time derivative of the quantity $\delta\chi$ is ill-defined at the moment τ_s at which

$$\frac{3}{2} \frac{G}{\tilde{G}} \frac{\bar{\rho}_c}{\bar{\rho}} - \frac{12\pi G\alpha}{a^{3(n+2)}} \left(3n+2 - 2 \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} \right) - \frac{8\pi\tilde{G}\alpha}{a^{3(n+2)}} \frac{k^2}{\mathcal{H}^2} = 0. \quad (78)$$

Provided that the quantities $\delta\dot{\chi}$ and $\dot{\Phi}$ are finite, the equation (77) determines $\Phi(\tau_s)$ as a function of $v_b(\tau_s)$, $\delta_b(\tau_s)$, $\delta\chi(\tau_s)$. Under the same assumption, taking the conformal time derivative of (75), (76) and performing some algebraic calculations, we can uniquely express $\delta\dot{\chi}(\tau_s)$, $\dot{\Phi}(\tau_s)$ in terms of these three quantities. Thus, the non-singularity of the solution at the moment τ_s requires extremely fine tuning of initial conditions.

The background equations yield $\mathcal{H} \approx 2/\tau$ in the dust dominated Universe (this equation becomes exact for $n = -2$ or $\alpha = 0$). Under this approximation, the equation (78) gives

$$k^2 \tau^2 = \frac{3}{4} \frac{G}{\tilde{G}} \frac{a^{3(n+2)}}{\pi \tilde{G} \alpha} \frac{\bar{\rho}_c}{\bar{\rho}} - 18 \frac{G}{\tilde{G}} (n+1). \quad (79)$$

The equation (79) has no solution at any k if and only if its right-hand side is negative. In this case the solution of the complete system of perturbed equations is free of singularities caused by ill-defined quantities $\delta\dot{\chi}$ and $\dot{\Phi}$.

At $\alpha \leq 0$, $n \geq -1$, the right-hand side of the equation (79) is negative and it has no solution. At $\alpha \leq 0$, $n < -1$, the condition of the absence of singularities takes the form

$$\frac{24\pi\tilde{G}|\alpha|}{a^{3(n+2)}} \leq \frac{1}{|n+1|} \frac{\bar{\rho}_c}{\bar{\rho}_b + \bar{\rho}_c}. \quad (80)$$

For positive α , the right side of the equation (79) can be negative only at $n > -1$ as long as

$$\frac{24\pi\tilde{G}\alpha}{a^{3(n+2)}} > \frac{1}{n+1} \frac{\bar{\rho}_c}{\bar{\rho}_b + \bar{\rho}_c}. \quad (81)$$

4.3 Instabilities in the radiation dominated Universe.

Given that the Universe contain dark matter and radiation, the analysis is very similar to the studied above case of dust dominance. Now, the total density perturbation $\delta\rho$ and the velocity potential v are given by

$$\delta\rho = \delta\rho_r - \beta\delta b, \quad (82)$$

$$(\bar{\rho} + \bar{p})v = \frac{4}{3}\bar{\rho}_r v_r + \beta\bar{b}\delta\dot{\chi}. \quad (83)$$

The position of the singular point of the system of perturbed equations is determined formally by the same equation (78).

Hereinafter we consider that the contribution of cold dark matter to the total energy density is small, i.e. $\bar{\rho}_c \ll \bar{\rho}$. We assume also that the higher derivative terms do not break the approximate equality $\mathcal{H} \approx 1/\tau$, which should be fulfilled at the radiation dominated stage. This allows us to rewrite the equation (78) as

$$k^2\tau^2 = \frac{3}{16} \frac{a^{3(n+2)}}{\pi\tilde{G}\alpha} \frac{G}{\tilde{G}} \frac{\bar{\rho}_c}{\bar{\rho}} - \frac{3}{2} \frac{G}{\tilde{G}} (3n+4). \quad (84)$$

At $\alpha \leq 0$, $n \geq -\frac{4}{3}$, the equation (84) has no solution. At $\alpha \leq 0$, $n < -\frac{4}{3}$, the solution does not exist under the condition

$$\frac{24\pi\tilde{G}|\alpha|}{a^{3(n+2)}} \leq \frac{1}{|n + \frac{4}{3}|} \frac{\bar{\rho}_c}{\bar{\rho}_r + \bar{\rho}_c}. \quad (85)$$

In the particular case $n = -2$, the equation (85) leads to the equation (43) due the inequality $\bar{\rho}_c/\bar{\rho}_r \ll 1$.

For positive α , the equation (84) has a solution only when $n > -\frac{4}{3}$ and there is the additional constraint

$$\frac{24\pi\tilde{G}\alpha}{a^{3(n+2)}} > \frac{1}{n + \frac{4}{3}} \frac{\bar{\rho}_c}{\bar{\rho}_r + \bar{\rho}_c}. \quad (86)$$

The equations (86) and (81), taken together, are incompatible with the equation (43) which holds at $n \neq -2$.

Now it is possible to summarize briefly some of our findings. The condition of absence of singularities in both radiation dominated and matter dominated epochs leads to the inequality (56) together with the constraint (43) at any power-law index n .

5 Evolution of perturbations.

Let us study the evolution of perturbations in a cosmological fluid with higher derivative corrections in more detail. To accomplish this, we consider two simple models. Using the results of the previous section, one can restrict the consideration to the case of non-positive coupling constant α .

5.1 Toy model.

In this Toy model, the Universe contain only dark matter with the action (27). The evolution of perturbations is governed by perturbed Einstein equations (66), (67). Using the background Einstein equations (39) and (40), one can write them as

$$\begin{aligned} \dot{\Phi} - \frac{8\pi\tilde{G}\alpha}{a^{3(n+2)}}k^2\delta\dot{\chi} = & - \left(\frac{5}{2} + \frac{1}{3}\frac{\tilde{G}}{G}\frac{k^2}{\mathcal{H}^2} - \frac{36\pi\tilde{G}\alpha}{a^{3(n+2)}}(n+2) \right) \mathcal{H}\Phi \\ & + \left(\frac{1}{2} - \frac{12\pi\tilde{G}\alpha}{a^{3(n+2)}}(n+2) \right) \mathcal{H}k^2\delta\chi, \end{aligned} \quad (87)$$

$$\begin{aligned} \ddot{\Phi} - \frac{3}{2} \left(1 - n\frac{24\pi\tilde{G}\alpha}{a^{3(n+2)}} + 2\frac{G}{\tilde{G}} \left(\frac{24\pi\tilde{G}\alpha}{a^{3(n+2)}} \right)^2 \right) \mathcal{H}^2\delta\dot{\chi} \\ = - \left(1 - 3\frac{G}{\tilde{G}}\frac{24\pi\tilde{G}\alpha}{a^{3(n+2)}} \right) \mathcal{H}\Phi - \frac{G}{\tilde{G}}\frac{24\pi\tilde{G}\alpha}{a^{3(n+2)}}\mathcal{H}k^2\delta\chi. \end{aligned} \quad (88)$$

The equations (87),(88) can be combined into a second order differential equation for the metric perturbations Φ . This equation has the form

$$f_1\ddot{\Phi} + f_2\dot{\Phi} + f_3\Phi = 0. \quad (89)$$

The coefficients f_1 , f_2 , f_3 are some functions of background quantities. Under condition (43), they are

$$f_1 = 1 - \frac{2}{9}\frac{k^2}{\mathcal{H}^2}\xi - \frac{8}{81}\frac{k^4}{\mathcal{H}^4}\xi^3, \quad (90)$$

$$f_2 = 3\mathcal{H} \left[1 + \frac{2}{27} (2 + 3n + (52 + 34n + 3n^2)\xi) \frac{k^2}{\mathcal{H}^2}\xi - \frac{8}{81}(3 + n)\frac{k^4}{\mathcal{H}^4}\xi^3 \right], \quad (91)$$

$$f_3 = -\mathcal{H}^2 \left[3(n+2)^2 + \frac{1}{9} ((3n+5)^2 + 4) \frac{k^2}{\mathcal{H}^2} + \frac{2}{9}\frac{k^4}{\mathcal{H}^4} + \frac{8}{81}\frac{k^6}{\mathcal{H}^6}\xi^2 \right] \xi, \quad (92)$$

where $\xi = 24\pi G\alpha/a^{3(n+2)}$.

At $\alpha = 0$, the equation (89) is reduced to

$$\ddot{\Phi} + \frac{6}{\tau}\dot{\Phi} = 0, \quad (93)$$

The general solution of this equation is

$$\Phi_{dust} = C_1 + \frac{C_2}{\tau^5}, \quad (94)$$

where C_1, C_2 are arbitrary constants. The constant solution corresponds to the growing adiabatic mode at the matter dominated stage [39]

$$\Phi = C_1, \quad \delta = -2C_1 - \frac{1}{6}C_1(k\tau)^2, \quad v = -\frac{1}{3}C_1k\tau, \quad (95)$$

where δ is the dust density contrast in the longitudinal gauge.

At $\alpha \neq 0$, the equation (89) can be solved in several special cases.

5.1.1 Short-wavelength perturbations.

Let us consider first the short-wavelength perturbations ($k \gg \mathcal{H}$), that satisfy to the additional condition

$$\frac{24\pi G |\alpha|}{a^{3(n+2)}} \frac{k^2}{\mathcal{H}^2} \ll 1. \quad (96)$$

On this scale, the equation (89) is reduced to the form

$$\ddot{\Phi} + 3\mathcal{H}\dot{\Phi} - \frac{16}{3}\mathcal{H}^2 \frac{\pi G \alpha}{a^{3(n+2)}} \frac{k^4}{\mathcal{H}^4} \Phi = 0. \quad (97)$$

To simplify the equation (97), one can use the approximation $a \approx \tau^2/\tau_0^2$, $\mathcal{H} \approx 2/\tau$. Then the condition (96) can be rewritten as

$$\frac{\pi G |\alpha| \tau_0^{6(n+2)}}{\tau^{2(3n+5)}} k^2 \ll 1. \quad (98)$$

The expression (98) shows the importance of the regime (96). At given k and $n \geq -5/3$, if the condition (98) is fulfilled at some time, it must be fulfilled at all subsequent times.

The characteristic time scale is equal to \mathcal{H}^{-1} . Consequently, the mass term in the equation (97) can be neglected when

$$\frac{24\pi G |\alpha|}{a^{3(n+2)}} \frac{k^4}{\mathcal{H}^4} \ll 1. \quad (99)$$

This fact is confirmed by the analysis of particular cases. The inequality (99) itself, within the above approximation, takes the form

$$\frac{\pi G |\alpha| \tau_0^{6(n+2)}}{\tau^{2(3n+4)}} k^4 \ll 1. \quad (100)$$

For example, at $n = 0$, the equation (97) takes the form

$$\ddot{\Phi} + \frac{6}{\tau}\dot{\Phi} - \frac{4\pi G \alpha}{3} \frac{k^4 \tau_0^{12}}{\tau^{10}} \Phi = 0. \quad (101)$$

The solution of this equation can be expressed in terms of Bessel functions J_ν and Y_ν [40]. The general solution has the form

$$\Phi_{(0)} = \frac{C_{1(0)}}{\tau^{5/2}} J_{-5/8} \left(\sqrt{-\frac{\pi G \alpha}{12}} \frac{\tau_0^6 k^2}{\tau^4} \right) + \frac{C_{2(0)}}{\tau^{5/2}} Y_{-5/8} \left(\sqrt{-\frac{\pi G \alpha}{12}} \frac{\tau_0^6 k^2}{\tau^4} \right), \quad (102)$$

where $C_{1(0)}$ and $C_{2(0)}$ are some constants. This expression oscillates and decreases rapidly when

$$\frac{\pi G |\alpha| \tau_0^{12} k^4}{\tau^8} \gg 1. \quad (103)$$

Later, under the condition (100), the general solution is not very different from the standard one (94), that can be checked by expanding the Bessel functions into Taylor series.

Let us consider now the perturbations with extremely short wavelength, such that

$$\frac{24\pi G |\alpha|}{a^{3(n+2)}} \frac{k}{\mathcal{H}} \gg 1. \quad (104)$$

At any fixed moment of time, provided that $\alpha \neq 0$, one can specify a small enough scale at which the condition above is fulfilled. The equation (89) takes the form

$$\ddot{\Phi} + 3(3+n)\mathcal{H}\dot{\Phi} + k^2\Phi = 0. \quad (105)$$

The solution of the equation (105) can be found by the WKB method. It has the form of oscillations which are damped at $n > -3$ or increased at $n < -3$. Since fast and unlimited growth (in the linear perturbation theory) of short-wavelength perturbations destroys the spatial homogeneity of the Universe, we obtain the constraint $n \geq -3$.

5.1.2 Long-wavelength perturbations.

On a large scale ($k \ll \mathcal{H}$), the equation (89) is reduced to

$$\ddot{\Phi} + 3\mathcal{H}\dot{\Phi} - \mathcal{H}^2 \left[3(n+2)^2 + \frac{1}{9} ((3n+5)^2 + 4) \frac{k^2}{\mathcal{H}^2} \right] \frac{24\pi G \alpha}{a^{3(n+2)}} \Phi = 0. \quad (106)$$

At $n = -2$, the long-wavelength solutions of this equation have the standard asymptotic behavior. At arbitrary n , by virtue of the equation (43), the mass term in this equation can be neglected. Then we obtain, with reasonable accuracy, the same equation for the metric perturbations as in the absence of higher derivative corrections.

The consideration of the Toy model shows that the characteristic tendency of the model is the suppression of short-wavelength perturbations. This feature, depending on the values of the parameters α and n , can lead to a cutoff of the power spectrum on cosmological scales. On the other hand, it may offer a way to explain the uniform distribution of the dark matter within the Solar system [41] by influence of higher derivative terms.

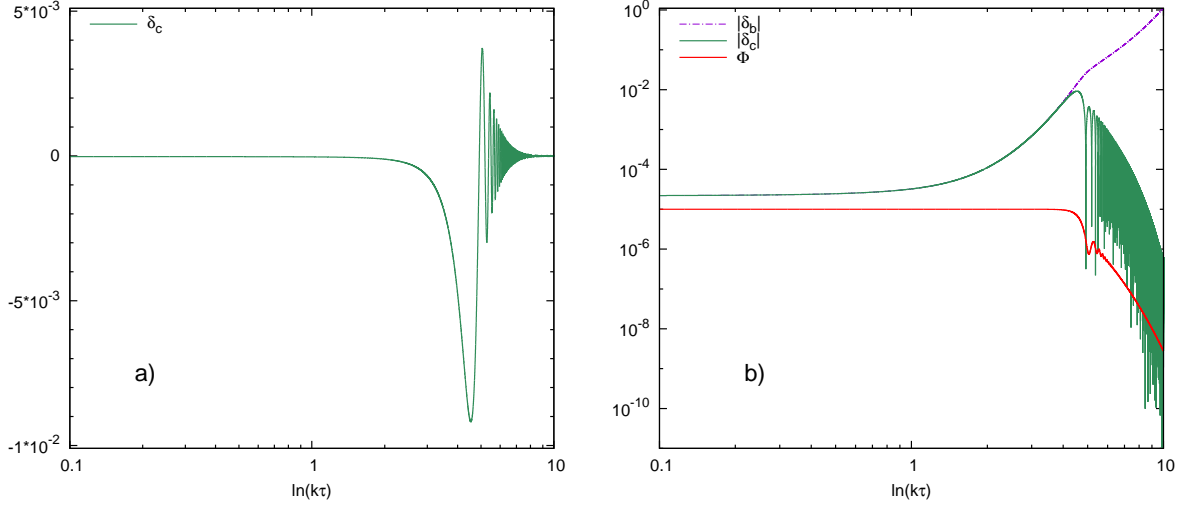


Figure 1: a) The evolution of the quantity $\delta_c \equiv -k^2\delta\chi + 3\Phi$, which is, at $\alpha = 0$, the dark matter density contrast in the longitudinal gauge. b) The evolution of quantities $|\delta_b|$, $|\delta_c|$, Φ on a logarithmic plot. The solution corresponds to $8\pi G\alpha = -10^{-6}$, $\frac{\bar{\rho}_c}{\bar{\rho}} = 0.8$, $\frac{\bar{\rho}_b}{\bar{\rho}} = 0.2$. The initial conditions are imposed at $\ln(k\tau_{in}) = -40$. Plots are normalized by $\Phi_0 = 10^{-5}$.

5.2 Matter dominated Universe.

Compared with the Toy model, more realistic models would include baryonic matter. In this subsection, we will study the evolution of perturbations in the cosmological model, briefly considered in Section 4.2.

In the longitudinal gauge, the Euler equation and the continuity equation for baryonic matter are [39, 42]

$$\dot{\delta}_b - 3\dot{\Phi} - k^2 v_b = 0, \quad (107)$$

$$\dot{v}_b + \mathcal{H}v_b - k^2\Phi = 0. \quad (108)$$

The equations (107), (108) combined with the perturbed Einstein equations (75), (76) form a complete set of equations. This system can be solved numerically by setting the initial adiabatic conditions on a large scale. At $n \geq -4/3$, due to imposed initial conditions, the condition (99) will be fulfilled during the entire evolution and the impact of the corrections on the solution of the system is negligible. At $-4/3 < n < -3$, the behaviour of the solution is more complex. We numerically analyze it only for the particular case $n = -2$.

It is convenient to introduce new variables $x = k\tau$, $u_b = kv_b$, $D_c = -k^2\delta\chi$, $D_b = \delta_b - 3\Phi$.

The governing equations when take the form

$$D'_b = u_b, \quad (109)$$

$$u'_b = -\frac{2}{x}u_b - \Phi, \quad (110)$$

$$\Phi' + 8\pi\tilde{G}\alpha D'_c = -\frac{1}{x} \left[\frac{\bar{\rho}_c}{\bar{\rho}} D_c + \frac{\rho_b}{\rho} D_b + \left(5 + \frac{1}{6} \frac{\tilde{G}}{G} x^2 \right) \Phi \right], \quad (111)$$

$$\begin{aligned} \Phi' + \frac{6}{x^2} \left(\frac{\bar{\rho}_c}{\bar{\rho}} + 24\pi G\alpha \left(1 + \frac{\bar{\rho}_c}{\bar{\rho}} \right) \right) D'_c &= -\frac{6}{x^2} \frac{G}{\tilde{G}} \frac{\bar{\rho}_b}{\rho} u_b \\ &+ \frac{2}{x} [(72\pi G\alpha - 1) \Phi + 24\pi G\alpha D_c]. \end{aligned} \quad (112)$$

We choose the usual adiabatic conditions, which corresponds to the growing adiabatic mode in case of zero corrections (95)

$$\Phi_{in} = \Phi_0, \quad D_{bin} = D_{cin} = -5\Phi_0 - \frac{1}{6}\Phi_0 x_{in}^2, \quad u_{bin} = -\frac{1}{3}\Phi_0 x_{in}. \quad (113)$$

The numerical solutions confirm the results of the above analytical consideration. On a large scale, the impact of the higher derivative terms is negligible. On a small scale, the dark matter perturbations are suppressed passing through several regimes. On a sufficiently small scale, the evolution of the metric perturbations is governed by the density contrast of the baryonic matter. The typical result (for $0 < -8\pi G\alpha \ll \bar{\rho}_c/\bar{\rho}$) is shown in Figure 1.

6 Conclusions and outlook.

We examined some simplest possible higher derivative corrections to the action of relativistic fluid within the effective field theory for hydrodynamics. Considered corrections can be described by two parameters, namely the coupling constant α and the power-law index n . Particular attention has been given to dark matter, described by the fluid action (27). The condition of absence of singularities in the background and perturbed solutions leads to the constraint $\alpha \leq 0$, as well as to the inequality $8\pi G\alpha/a^{3(n+2)} \ll 1$ that should be fulfilled during the radiation dominated and the matter dominated stages. The evolution of perturbations has been investigated in two models of the matter dominated Universe. We obtain what considered higher derivative corrections have a strong influence on the evolution of short-wavelength perturbations. Consideration of the Toy model (without baryonic matter) shows that the perturbations are suppressed in the short-wavelength limit at $n > -3$. The numerical analysis in a more realistic model with baryonic matter confirms and refines results of the analytical treatment. In this model, there is a wide range of parameters in which the dark matter can be considered as homogeneous on a sufficiently small scale.

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